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HEATED JET DISCHARGED VERTICALLY INTO AMBIENTS OF UNIFORM AND LINEAR TEMPERATURE PROFILES

FUMIMARU OGINO, HIROMI TAKEUCHI, ISAO KUDO and TOKURO MIZUSHINA
Department of Chemical Engineering, Kyoto University, Kyoto, Japan

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Abstract—The same scaling law as that proposed by Chen and Rodi for correlating the centerline velocity and temperature of a buoyant jet discharged into a uniform temperature ambient is derived from the dimensional analysis of the general governing equations and boundary conditions, and compared with the experimental measurements.

The functional forms of the centerline velocity and temperature, zero momentum and zero buoyancy heights for the buoyant jet discharged into a linearly stratified ambient are deduced by the same analysis as that for uniform temperature ambient and confirmed by the experiment. In particular, the zero momentum and zero buoyancy heights are correlated experimentally with the product of discharge Froude number and the temperature gradient of the ambient fluid.

NOMENCLATURE

d_0, discharge nozzle diameter [m];
Fr, discharge Froude number, $U_0^2/\beta(T_0 - T_{\infty 0})d_0 g$;
g, gravitational acceleration [m s^{-2}];
r, radial coordinate [m];
r^*, dimensionless radial coordinate, r/d_0;
Re, Reynolds number, $\rho_0 U_0 d_0/\mu_0$;
S, dimensionless temperature gradient of ambient fluid, $[d_0/(T_0 - T_{\infty 0})](dT_{\infty x}/dx)$;
T, time-averaged temperature [K];
T^*, dimensionless time-averaged temperature, $(T - T_{\infty x})/(T_0 - T_{\infty 0})$;
U, time-averaged velocity component in x-direction [m s^{-1}];
U^*, dimensionless time-averaged velocity, U/U_0;
u, fluctuating velocity component in x-direction [m s^{-1}];
u^*, dimensionless fluctuating velocity, u/U_0;
V, time-averaged velocity component in r-direction [m s^{-1}];
V^*, dimensionless time-averaged velocity, V/U_0;
v, fluctuating velocity component in r-direction [m s^{-1}];
v^*, dimensionless fluctuating velocity, v/U_0;
X, vertical coordinate from virtual source [m];
X^*, dimensionless vertical coordinate, X/d_0;
X_M, zero momentum height [m];
X_T, zero buoyancy height [m];
x, vertical coordinate [m];
x^*, dimensionless vertical coordinate, x/d_0.

Greek symbols

β, coefficient of volume expansion [K^{-1}];
θ, fluctuating temperature [K];
θ^*, dimensionless fluctuating temperature, $\theta/(T_0 - T_{\infty 0})$;
μ, viscosity [Pa · s];
ρ, density [kg m^{-3}].

Subscripts

m, value at jet centerline;
0, value at jet discharge;
∞, ambient value;
∞x, ambient value at x.

Superscript

#, dimensionless quantities, equations (9)–(11).

1. INTRODUCTION

ONE OF the flow configurations important in the environmental heat-transfer processes with significant buoyancy effects is a vertical buoyant jet discharged into a stagnant environment.

A vertical buoyant jet discharged into a uniform temperature ambient behaves like a momentum jet in the region near the discharging nozzle since the momentum force dominates the flow. There follows an intermediate region where the influence of the initial momentum force becomes smaller. In the final region or far field the buoyancy force completely dominates the flow and the buoyant jet behaves like a plume. Both momentum jet and pure plume will attain a self-preserving profile at some distance from the discharging nozzle, but with a different similarity profile and different scaling law for decay of the flow. Therefore, the experimental data of general buoyant jet cannot be correlated with a single curve from the region near the discharging point where it behaves like a momentum jet to the far field where it behaves like a plume.

Although a number of investigations on vertical buoyant jet are available in literatures [1–8], these results often appear confusing because different investigators used different scaling parameters. Recently, Chen and Rodi [9] proposed a unified correlation of the decay of a vertical buoyant jet in a uniform temperature ambient using a dimensional consideration and showed to be applicable for both momentum jets and plumes as well as general buoyant jets.

In this paper the same scaling law as Chen and Rodi's is derived from the more fundamental procedure, that is, the dimensional analysis of the governing equations and the boundary conditions for the vertical round buoyant jet in a uniform temperature ambient.

If a heated jet is discharged into a stably stratified environment, the mixing of the jet fluid and the ambient fluid makes the jet denser and eventually the buoyancy force acting on the jet will change sign to become negative because the density of the ambient fluid decreases steadily upwards. Then the jet will stop rising when the upward momentum vanishes by the action of the negative buoyancy force. Thus the buoyant jet in a stably stratified ambient has a finite height of rise and will then spread out horizontally in a thin layer.

Both the height at which buoyancy force goes to zero, called the zero buoyancy height, and the height at which momentum reduces to zero, called the zero momentum height, are important unknowns to be determined.

In the present paper, the functional dependence of these heights on the stratification of ambient fluid is deduced by the same analyzing method as that for a uniform temperature ambient, and the correlation equations of them obtained experimentally for the buoyant jets in a stratified ambient with a linear temperature profile are presented.

2. ANALYSIS

Figure 1 shows the coordinate system for axisymmetric buoyant jet of temperature T_0 and velocity U_0 discharged into an ambient with uniform temperature of T_∞ along with typical examples of measured temperature of the jet centerline and the ambient fluid.

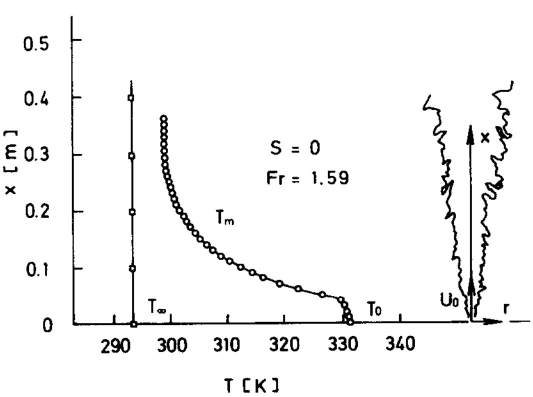

FIG. 1. Schematic representation of a buoyant jet in a uniform temperature ambient.

Under the assumption that the flow is steady and of the boundary layer type, and employing the Boussinesq approximation, in which the density variation is accounted for only in the buoyancy force term, the governing equations for the velocity and temperature can be written in the dimensionless form as,

$$\frac{\partial U^*}{\partial x^*} + \frac{1}{r^*}\frac{\partial}{\partial r^*}(r^*V^*) = 0 \tag{1}$$

$$U^*\frac{\partial U^*}{\partial x^*} + V^*\frac{\partial U^*}{\partial r^*} = \frac{1}{r^*}\frac{\partial}{\partial r^*} \times (-r^*\overline{u^*v^*}) + \frac{1}{Fr}T^* \tag{2}$$

$$U^*\frac{\partial T^*}{\partial x^*} + V^*\frac{\partial T^*}{\partial r^*} = \frac{1}{r^*}\frac{\partial}{\partial r^*}(-r^*\overline{v^*\theta^*}). \tag{3}$$

The viscous and conductive terms are neglected in equations (2) and (3) since they are small compared with the turbulent stress $-\overline{u^*v^*}$ and heat flux $-\overline{v^*\theta^*}$. Furthermore the viscous dissipation is neglected in equation (3).

The boundary conditions are:

B.C.1 $x^* = 0,$

$$\left.\begin{array}{lll} 0 \leqq r^* < \frac{1}{2}; & U^* = 1 & T^* = 1 \\ \frac{1}{2} \leqq r^*; & U^* = 0, & T^* = 0 \end{array}\right\} \tag{4}$$

B.C.2 $$r^* = 0; \quad \frac{\partial U^*}{\partial r^*} = 0, \quad \frac{\partial T^*}{\partial r^*} = 0, \quad V^* = 0 \tag{5}$$

B.C.3 $$r^* = \infty; \quad U^* = 0, \quad T^* = 0. \tag{6}$$

If equations (1)–(3) could be solved for U^* and T^* with the boundary conditions (4)–(6) on the assumption that the turbulent stress and heat flux are given as some known functions of x^* and r^*, the solutions would in principle be of the form

$$U^* = U^*(x^*, r^*; Fr) \tag{7}$$

$$T^* = T^*(x^*, r^*; Fr). \tag{8}$$

That is, the functional dependence will include all the reduced variables and the one dimensionless number appearing in the differential equations. No additional dimensionless groups enter via the foregoing boundary conditions.

Although equations (7) and (8) are exact solutions, the inclusion of the Froude number in the functional form is inconvenient for the present purpose to give a unified correlation of the decay of turbulent buoyant jet. Therefore, we define the following new variables:

$$\begin{array}{ll} U^\# = AU^*, & V^\# = AV^*, \\ u^\# = Au^*, & v^\# = Av^* \end{array} \tag{9}$$

$$T^\# = BT^*, \quad \theta^\# = B\theta^* \tag{10}$$

$$x^\# = Cx^*, \quad r^\# = Cr^* \tag{11}$$

where A, B and C are functions of Fr to be determined.

Substituting equations (9)–(11) into equations (1)–(3) and the boundary conditions (4)–(6), we obtain

$$\frac{\partial U^{\#}}{\partial x^{\#}}+\frac{1}{r^{\#}}\frac{\partial(r^{\#}U^{\#})}{\partial r^{\#}}=0 \tag{12}$$

$$U^{\#}\frac{\partial U^{\#}}{\partial x^{\#}}+V^{\#}\frac{\partial U^{\#}}{\partial r^{\#}}=\frac{1}{r^{\#}}\frac{\partial}{\partial r^{\#}}\times(-r^{\#}\overline{u^{\#}v^{\#}})+\frac{A^2}{BCFr}T^{\#} \tag{13}$$

$$U^{\#}\frac{\partial T^{\#}}{\partial x^{\#}}+V^{\#}\frac{\partial T^{\#}}{\partial r^{\#}}=\frac{1}{r^{\#}}\frac{\partial}{\partial r^{\#}}(-r^{\#}\overline{v^{\#}\theta^{\#}}) \tag{14}$$

B.C.1 $x^{\#}=0$,

$$\begin{aligned}&0\leqq r^{\#}<\frac{C}{2}; \quad U^{\#}=A, \quad T^{\#}=B\\ &\frac{C}{2}\leqq r^{\#}; \quad U^{\#}=0, \quad T^{\#}=0\end{aligned} \tag{15}$$

B.C.2 $r^{\#}=0;\quad \dfrac{\partial U^{\#}}{\partial r^{\#}}=0,\quad \dfrac{\partial T^{\#}}{\partial r^{\#}}=0,\quad V^{\#}=0$ (16)

B.C.3 $r^{\#}=\infty;\quad U^{\#}=0,\quad T^{\#}=0.$ (17)

If we put $A^2/BCFr=1$ in equation (13), the solutions for $U^{\#}$ and $T^{\#}$ include no parameter from the differential equations, but A, B and C enter into them via the boundary condition at $x^{\#}=0$. Hence, we consider the limiting form of the solutions for large $x^{\#}$ where the boundary condition at $x^{\#}=0$ is not important and replace B.C.1 by

B.C.1′ $$\int_0^{\infty}U^{\#}T^{\#}r^{\#}\,\mathrm{d}r^{\#}=\frac{ABC^2}{8}. \tag{18}$$

Furthermore, we use the following relations for the limiting case of $Fr\to\infty$, that is, for the momentum jet, as a condition to be satisfied by the solutions.

$$\begin{aligned}Fr\to\infty \quad & U^{\#}\to\text{numerical const.}\cdot\frac{AC}{x^{\#}}\\ & T^{\#}\to\text{numerical const.}\cdot\frac{BC}{x^{\#}}\end{aligned} \tag{19}$$

These are the counterparts of the choice of $A^2/BCFr=1$.

Now, if we assume ABC^2 in equation (18) and AC, BC in equation (19) to be unity, we obtain $A=\sqrt{Fr}$, $B=\sqrt{Fr}$ and $C=1/\sqrt{Fr}$. Thus the limiting forms of the solutions for $U^{\#}$ and $T^{\#}$ at large $x^{\#}$ are:

$$U^{\#}=U^{\#}(x^{\#},r^{\#}) \tag{20}$$

$$T^{\#}=T^{\#}(x^{\#},r^{\#}) \tag{21}$$

where $U^{\#}=\sqrt{Fr}(U/U_0)$, $T^{\#}=\sqrt{Fr}[(T-T_\infty)/(T_0-T_\infty)]$ and $x^{\#}=x/d_0\sqrt{Fr}$, $r^{\#}=r/d_0\sqrt{Fr}$. From equations (20) and (21) the decay of centerline velocity and temperature in a round buoyant jet can then be described by

$$\sqrt{Fr}\frac{U_m}{U_0}=\text{func.}\left(\frac{x}{d_0\sqrt{Fr}}\right) \tag{22}$$

$$\sqrt{Fr}\frac{T_m-T_\infty}{T_0-T_\infty}=\text{func.}\left(\frac{x}{d_0\sqrt{Fr}}\right). \tag{23}$$

In Fig. 2 a buoyant jet discharged into a linearly stratified ambient is schematically sketched along with typical examples of measured temperature of the jet centerline and the ambient fluid. For the governing equations of this case, an additional term $-SU^*$ must be added on the RHS of equation (3), where S denotes the dimensionless temperature gradient of the ambient fluid. The remaining equations (1) and (2), and the boundary conditions (4)–(6) are valid also for such a stratified condition. Applying the transformation given by equations (9)–(11) with $A=B=\sqrt{Fr}$ and $C=1/\sqrt{Fr}$, we can obtain the solutions for $U^{\#}$ and $T^{\#}$ as,

$$U^{\#}=U^{\#}(x^{\#},r^{\#};\,SFr) \tag{24}$$

$$T^{\#}=T^{\#}(x^{\#},r^{\#};\,SFr). \tag{25}$$

Thus the centerline velocity and temperature can be described by

$$\sqrt{Fr}\frac{U_m}{U_0}=\text{func.}\left(\frac{x}{d_0\sqrt{Fr}};\,SFr\right) \tag{26}$$

$$\sqrt{Fr}\frac{T_m-T_{\infty x}}{T_0-T_{\infty 0}}=\text{func.}\left(\frac{x}{d_0\sqrt{Fr}};\,SFr\right). \tag{27}$$

The zero momentum and zero buoyancy heights can be obtained by putting $U_m=0$ and $T_m=T_{\infty x}$ in equations (26) and (27) as,

$$\frac{x_M}{d_0\sqrt{Fr}}=\text{func.}(SF_r) \tag{28}$$

$$\frac{x_T}{d_0\sqrt{Fr}}=\text{func.}(SFr). \tag{29}$$

3. EXPERIMENTAL APPARATUS AND PROCEDURE

The experimental apparatus is shown schematically in Fig. 3. The test tank 1 of 2 m long, 1 m wide and 1 m deep was made of 2.0 cm thick plain transparent acrylic plates.

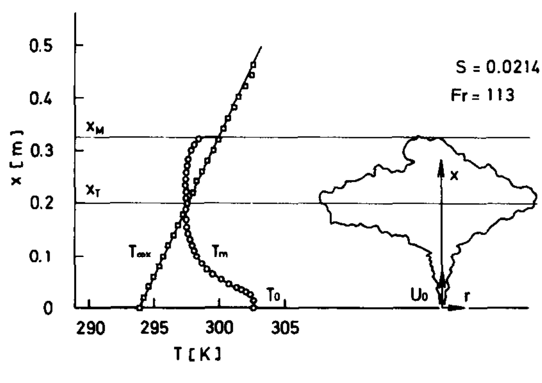

FIG. 2. Schematic representation of a buoyant jet in a stratified ambient.

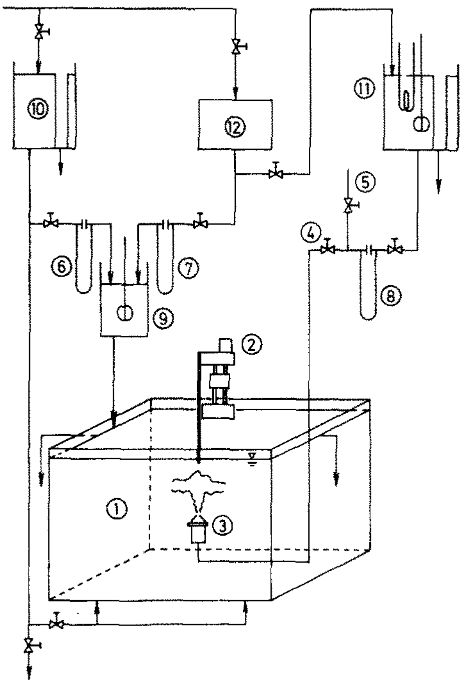

FIG. 3. Schematic drawing of the experimental apparatus: (1) test tank; (2) traverser; (3) contraction nozzle; (4) needle valve; (5) tracer inlet; (6–8) orifice meter; (9) mixing tank; (10, 11) head tank; (12) heater.

Filtered city water was used as ambient fluid and its temperature was kept uniform between 285 K and 300 K for all experiments of buoyant jet in uniform temperature environment. For experiments with stratified ambients, the linear temperature profile was achieved by filling the test tank layer by layer with water of different temperatures for the mixing tank 9. The resulting temperature profiles were found to be smoothed out in 2 h after the tank was filled with fluid. A typical measured temperature profile is shown in Fig. 2.

For jet flows, two contraction nozzles of different diameters, 4.9 and 10.2 mm, were used to cover a wide range of Froude number. The jet fluid was also city water which was heated in the heater 12 and of which temperature was regulated at 300 K $\sim$ 350 K in the tank 11. Flow rate was measured by a calibrated orifice meter 8.

A hot-film probe (TSI 1210–60 W) was used to measure the centerline velocity of the buoyant jets. The output signal from anemometer (DISA 55M10) was corrected by using the temperature-velocity calibration of the probe.

The centerline temperature was measured by a chromel-alumel thermocouple (Phillips 2ABAc025TI). The hot-film probe and the thermocouple were traversed vertically with a constant speed, 0.6 cm s^{-1}, and the measurements of the velocity and temperature were repeated at least 2 times under the same condition to obtain accurate time-averaged values.

As shown in Fig. 2 the zero momentum and zero buoyancy heights were determined as the position where the jet centerline temperature jumped discontinuously and the position where it was equal to the ambient temperature respectively. These heights were also measured photographically by adding the India ink to the jet fluid from the tracer inlet 5, that is, the zero momentum height was determined as the terminal height of rise and the position of zero buoyancy was assumed to be the level where the jet spread horizontally.

Reynolds numbers at discharging point ranged from 1.4×10^3 to 1.1×10^4 and the discharge Froude numbers were 1.0 to 8.0×10^3 for the experiments with the uniform temperature ambients. The measurements in the stably stratified ambients were performed for Reynolds numbers ranging from 2.0×10^3 to 6.2×10^3, Froude numbers $1.4\text{–}3.2 \times 10^4$ and the dimensionless temperature gradients of ambient fluid from 3.3×10^{-3} to 1.5×10^{-1}.

4. EXPERIMENTAL RESULTS AND DISCUSSION

Figures 4 and 5 show respectively the measured centerline velocities and temperatures of buoyant jets in uniform temperature ambients. The coordinates are chosen in accordance with equations (22) and (23) except that X, the vertical distance from the virtual source, instead of x is used in order to compare the present experimental results in the momentum jet region with the correlations for the pure momentum jet obtained by various investigators such as Keagy *et al.* [11] and Wygnanski *et al.* [12]. The virtual source was determined from a plot of T^{*-1} vs x^* at the region of $4 < x^* < 20$ as the intercept of the resulting line and the x^* coordinate.

In Fig. 4 open symbols represent the results obtained by using a Pitot tube. The data points of both velocity and temperature seem to fall closely on a single curve, thus confirming the relations given by equations (22) and (23).

For comparison the correlation curves presented by Chen and Rodi [9] are depicted by broken lines. The Chen and Rodi's correlation for centerline temperature was obtained from the experimental results of Abraham [1], Pryputniewicz and Bowley [5] and Hayashi and Ito [10]. On the other hand, the curve for velocity represents an approximate relationship derived by connecting the limiting curves for momentum jet and plume because of lack of the available velocity measurements.

The present experimental result for centerline temperature agrees very well with the Chen and Rodi's correlation, whereas the data for velocity are consistently lower than the curve of Chen and Rodi. This disagreement may be due to the adoption of somewhat higher values of decay constants, 6.2 for momentum jet and 4.7 for plume in the correlation of Chen and Rodi. The present experimental result shows that the decay constants are 5.8 for the momentum jet region and 3.4 for the plume region. Keagy *et al.* [11] presented the

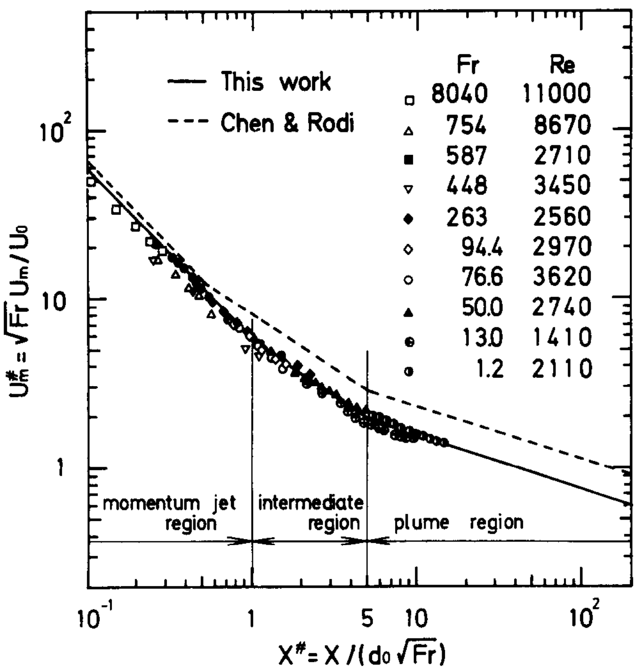

FIG. 4. Decay of jet centerline velocity in a uniform temperature ambient.

decaý constant for the momentum jet to be 5.8 and Wygnanski and Fiedler [12] implied 5.7. The constant for the plume is proposed to be 3.4 by George, Alpert and Tamanini [13] and 3.29 by Nakagome and Hirata [14]. The present experimental constants are in good agreement with these values.

The following correlation equations are obtained for the decay of the centerline velocity and temperature in buoyant jets discharged into uniform temperature ambients

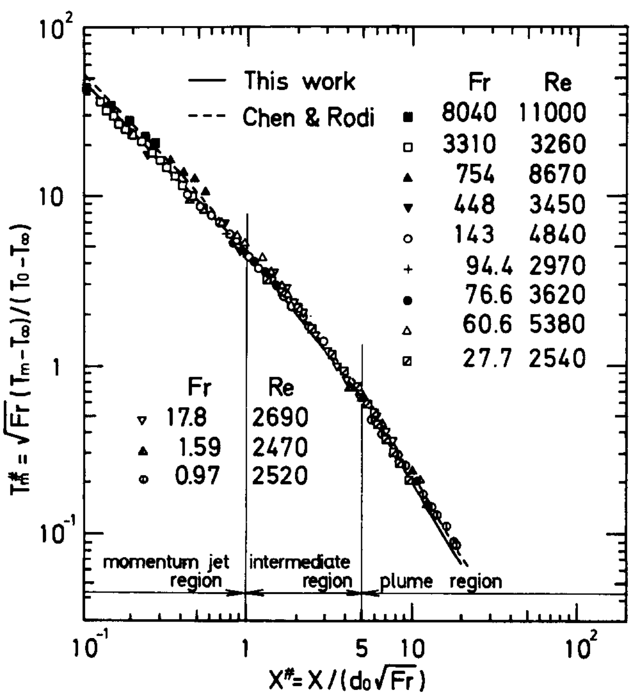

FIG. 5. Decay of jet centerline temperature in a uniform temperature ambient.

$$\frac{X}{d_0\sqrt{Fr}} \leqq 1.0$$

$$\sqrt{Fr}\frac{U_m}{U_0} = 5.8\left(\frac{X}{d_0\sqrt{Fr}}\right)^{-1} \quad (30)$$

$$\sqrt{Fr}\frac{T_m - T_\infty}{T_0 - T_\infty} = 4.8\left(\frac{X}{d_0\sqrt{Fr}}\right)^{-1} \quad (31)$$

$$1.0 \leqq \frac{X}{d_0\sqrt{Fr}} \leqq 5.0$$

$$\sqrt{Fr}\frac{U_m}{U_0} = 5.8\left(\frac{X}{d_0\sqrt{Fr}}\right)^{-2/3} \quad (32)$$

$$\sqrt{Fr}\frac{T_m - T_\infty}{T_0 - T_\infty} = 4.8\left(\frac{X}{d_0\sqrt{Fr}}\right)^{-5/4} \quad (33)$$

$$5.0 \leqq \frac{X}{d_0\sqrt{Fr}}$$

$$\sqrt{Fr}\frac{U_m}{U_0} = 3.4\left(\frac{X}{d_0\sqrt{Fr}}\right)^{-1/3} \quad (34)$$

$$\sqrt{Fr}\frac{T_m - T_\infty}{T_0 - T_\infty} = 9.4\left(\frac{X}{d_0\sqrt{Fr}}\right)^{-5/3}. \quad (35)$$

Figures 6 and 7 show respectively the measured centerline velocities and temperatures for the buoyant jets discharged into the linearly stratified ambients. Although the data points are somewhat scattering, the results seem to confirm equations (26) and (27), that is, $\sqrt{Fr}(U_m/U_0)$ and $\sqrt{Fr}[(T_m - T_{\infty x})/(T_0 - T_{\infty x})]$ are functions of $X/d_0\sqrt{Fr}$ and SFr only. As seen from Figs. 6 and 7, the centerline velocity and temperature deviate from those for uniform temperature ambient at smaller value of $x^{\#}$ with increasing the stratification parameter, SFr.

Figure 8 shows the zero momentum heights plotted against the stratification parameter SFr. The results obtained from photographs are also plotted along with the data of Fox [15], Fan [16], Abraham and Eysink [17], and Crawford and Leonard [18]. All data

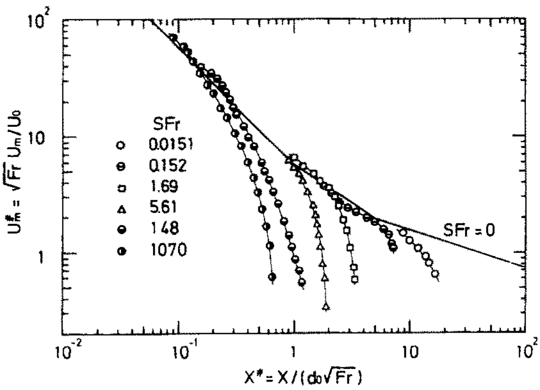

FIG. 6. Decay of jet centerline velocity in a linearly stratified ambient.

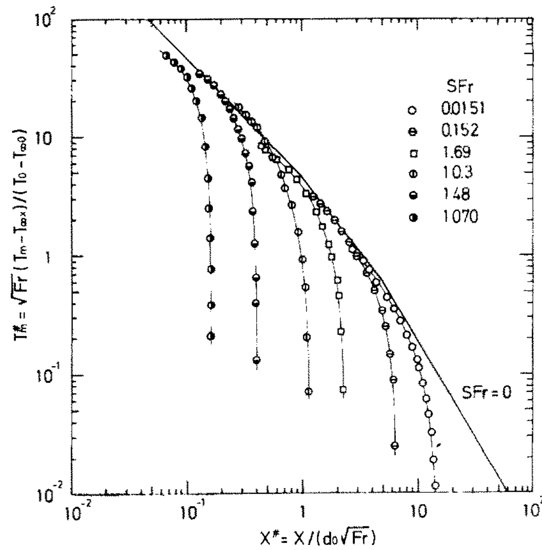

FIG. 7. Decay of jet centerline temperature in a linearly stratified ambient.

fall closely on a single line, thus confirming the validity of equation (28), and the correlation line can be described by

$$\frac{X_M}{d_0\sqrt{Fr}} = 4.80(SFr)^{-1/3}(10^{-3} < SFr < 10^3). \quad (36)$$

This means that the maximum height X_M of buoyant jets discharged into a linearly stratified ambient is proportional to $(dT_{\infty x}/dx)^{-1/3}$ and $(Fr)^{1/6}$. The correlation proposed by Morton *et al.* [7] can be rewritten in terms of the present notation as

$$\frac{X_M}{d_0\sqrt{Fr}} = 4.21(SFr)^{-3/8}. \quad (37)$$

For comparison, equation (37) is plotted in Fig. 8 by a broken line.

The zero buoyancy heights are depicted in Fig. 9, and can be correlated by the following equation

$$\frac{X_T}{d_0\sqrt{Fr}} = 2.90(SFr)^{-2/5}(10^{-2} < SFr < 10^3). \quad (38)$$

This equation shows that the zero buoyancy height X_T is proportional to $(dT_{\infty x}/dx)^{-2/5}$ and $(Fr)^{1/10}$.

If we assume that equation (38) can be extrapolated to the region of $SFr < 10^{-2}$, the unreasonable result evolves, that is, the zero buoyancy height becomes equal to the zero momentum height at $SFr = 5.22 \times 10^{-4}$. Therefore some different correlation should be applied to such a region of small value of SFr which could not be covered by the present experiment.

5. CONCLUSIONS

(1) The centerline velocity and temperature for the buoyant jet discharged into uniform temperature ambient are described by

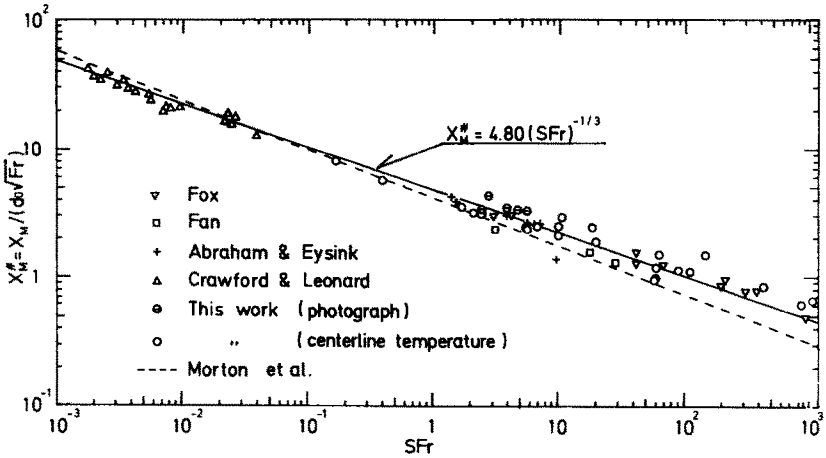

FIG. 8. Correlation of zero momentum height with stratification parameter.

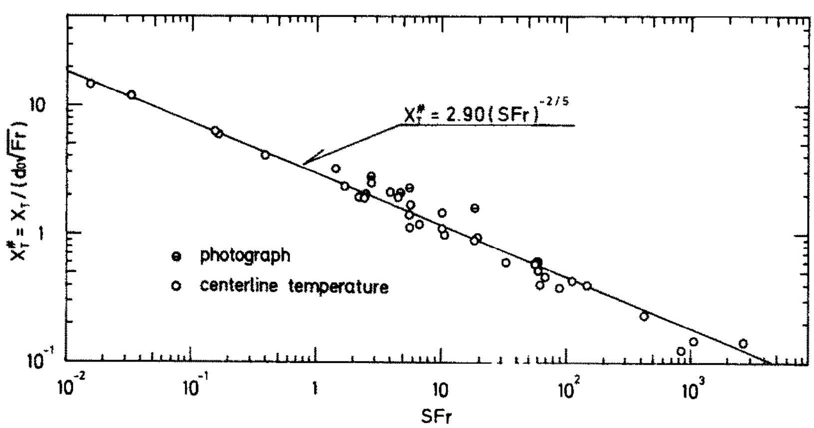

FIG. 9. Correlation of zero buoyancy height with stratification parameter.

$$\frac{X}{d_0\sqrt{Fr}} \leqq 1.0$$

$$\sqrt{Fr}\frac{U_m}{U_0} = 5.8\left(\frac{X}{d_0\sqrt{Fr}}\right)^{-1}$$

$$\sqrt{Fr}\frac{T_m - T_\infty}{T_0 - T_\infty} = 4.8\left(\frac{X}{d_0\sqrt{Fr}}\right)^{-1}$$

$$1.0 \leqq \frac{X}{d_0\sqrt{Fr}} \leqq 5.0$$

$$\sqrt{Fr}\frac{U_m}{U_0} = 5.8\left(\frac{X}{d_0\sqrt{Fr}}\right)^{-2/3}$$

$$\sqrt{Fr}\frac{T_m - T_\infty}{T_0 - T_\infty} = 4.8\left(\frac{X}{d_0\sqrt{Fr}}\right)^{-5/4}$$

$$5.0 \leqq \frac{X}{d_0\sqrt{Fr}}$$

$$\sqrt{Fr}\frac{U_m}{U_0} = 3.4\left(\frac{X}{d_0\sqrt{Fr}}\right)^{-1/3}$$

$$\sqrt{Fr}\frac{T_m - T_\infty}{T_0 - T_\infty} = 9.4\left(\frac{X}{d_0\sqrt{Fr}}\right)^{-5/3}.$$

(2) The functional dependences of the centerline velocity and temperature of the buoyant jet discharged into linearly stratified ambient on the discharge Froude number and the temperature gradient of ambient fluid are expressed as

$$\sqrt{Fr}\frac{U_m}{U_0} = \text{func.}\left(\frac{X}{d_0\sqrt{Fr}},\ SFr\right)$$

$$\sqrt{Fr}\frac{T_m - T_{\infty x}}{T_0 - T_{\infty 0}} = \text{func.}\left(\frac{X}{d_0\sqrt{Fr}},\ SFr\right).$$

(3) The zero momentum and zero buoyancy heights are correlated by the following equations

$$\frac{X_M}{d_0\sqrt{Fr}} = 4.80(SFr)^{-1/3}(10^{-3} < SFr < 10^3)$$

$$\frac{X_T}{d_0\sqrt{Fr}} = 2.90(SFr)^{-2/5}(10^{-2} < SFr < 10^3).$$

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JETS CHAUDS EN DETENTE VERTICALE DANS UNE AMBIANCE ISOTHERME OU A PROFIL LINEAIRE DE TEMPERATURE

Résumé—La même loi d'échelle que celle proposée par Chen et Rodi pour la vitesse et la température le long de l'axe d'un panache dans une ambiance à température uniforme est tirée de l'analyse dimensionnelle des équations générales et des conditions aux limites et elle est comparée avec les résultats expérimentaux.

Les formes fontionnelles de la température et de la vitesse le long de l'axe, les hauteurs de quantité de mouvement et d'élévation du jet se détendant dans une ambiance linéairement stratifiée sont déduites, par la même analyse, comme pour l'ambiance isotherme et sont confirmées par l'expérience. En particulier, les hauteurs sont exprimées en fonction du produit du nombre de Froude de décharge et du gradient de température du fluide ambiant.

EIN ERHITZTER FREISTRAHL, DER SENKRECHT IN EINE UMGEBUNG VON GLEICHFÖRMIGEN UND LINEAR VERÄNDERLICHEN TEMPERATURPROFILEN AUSSTRÖMT

Zusammenfassung—Das von Chen und Rodi zur Korrelation von Geschwindigkeit und Temperatur in der Achse eines aufsteigenden, in eine Umgebung mit gleichförmiger Temperatur ausströmenden Strahls formulierte Ähnlichkeitsgesetz wird durch Dimensionsanalyse aus den allgemeinen Grundgleichungen und Randbedingungen entwickelt und mit experimentellen Meßwerten verglichen.

Die funktionellen Zusammenhänge von Geschwindigkeit und Temperatur in der Achse sowie der Höhen, bei denen Impuls und Auftrieb null werden, werden für den aufsteigenden, in eine linear geschichtete Umgebung ausströmenden Strahl auf dieselbe Weise abgeleitet wie für die Umgebung mit gleichförmiger Temperatur. Die Ergebnisse werden experimentell bestätigt. Im einzelnen werden die Höhen, bei denen Impuls bzw. Auftrieb null werden, experimentell mit dem Produkt von Abström-Froude-Zahl und Temperaturgradient im umgebenden Fluid korreliert.

ВЕРТИКАЛЬНОЕ ИСТЕЧЕНИЕ НАГРЕТОЙ СТРУИ В ОКРУЖАЮЩУЮ СРЕДУ С РАВНОМЕРНЫМИ И ЛИНЕЙНЫМИ ПРОФИЛЯМИ ТЕМПЕРАТУР

Аннотация — Методом анализа размерностей выведено уравнение подобия, аналогичное уравнению, предложенному Ченом и Роди для описания скорости и температуры на оси свободноконвективной струи, истекающей в окружающую среду с постоянной температурой. Проведено сравнение с результатами экспериментов.

Функциональные зависимости для скорости и температуры на оси струи, нулевого импульса и нулевых высот свободноконвективной струи, истекающей в линейно стратифицированную окружающую среду, выведены с помощью того же анализа, что и для случая окружающей среды с постоянной температурой. Получено экспериментальное подтверждение результатов. В частности, существует функциональная связь между экспериментальными значениями нулевого импульса и нулевой высоты подъема струи и произведением числа Фруда для истечения на градиент температуры окружающей жидкости.